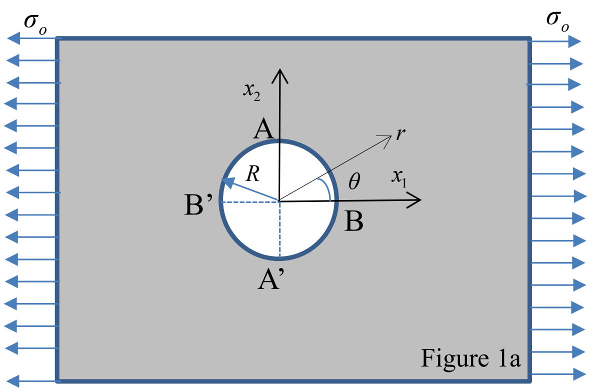
Mechanics of Solids: Stress Concentration

Large plate with a circular hole under uniaxial load (stress concentration)



$$\sigma_{rr} = \frac{\sigma_o}{2}\left(1 - \frac{R^2}{r^2}\right) + \frac{\sigma_o}{2}\left(1 + \frac{3R^4}{r^4} - \frac{4R^2}{r^2}\right)\cos 2\theta$$

$$\sigma_{\theta\theta} = \frac{\sigma_o}{2}\left(1 + \frac{R^2}{r^2}\right) - \frac{\sigma_o}{2}\left(1 + \frac{3R^4}{r^4}\right)\cos 2\theta$$

$$\sigma_{r\theta} = -\frac{\sigma_o}{2}\left(1 - \frac{3R^4}{r^4} + \frac{2R^2}{r^2}\right)\sin 2\theta$$

Stress at points A, A': $\sigma_{\theta\theta} = 3\sigma_o$
Stress at points B, B': $\sigma_{\theta\theta} = -\sigma_o$

The problem is solved in two phases:

Phase 1: we consider the plate without a hole and a stress function,

$$\Phi(r,\theta) = \frac{1}{2}\sigma_0 r^2 \sin^2\theta$$

The resulting stresses are:

$$\sigma_{rr} = \frac{1}{2}\sigma_0(1+\cos 2\theta);\quad \sigma_{\theta\theta} = \frac{1}{2}\sigma_0(1-\cos 2\theta);\quad \sigma_{r\theta} = -\frac{1}{2}\sigma_0 \sin 2\theta$$

Phase 2: we consider the plate with the hole and the stress function

$$\Phi(r,\theta) = f_1(r) + f_2(r) r^2 \cos 2\theta$$

Functions $f_1(r), f_2(r)$ are identified.

Boundary conditions.

$$\sigma_{rr}\big|_{r=R} = \sigma_{r\theta}\big|_{r=R} = 0 \qquad \sigma_{rr}\big|_{r\to\infty} = \frac{1}{2}\sigma_0(1+\cos 2\theta)$$

$$\sigma_{\theta\theta}\big|_{r\to\infty} = \frac{1}{2}\sigma_0(1-\cos 2\theta);\quad \sigma_{r\theta}\big|_{r\to\infty} = -\frac{1}{2}\sigma_0 \sin 2\theta$$